The mathematics of juggling

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Asynchronous siteswap notation

Rules :

- at each unit of time, exactly one ball is thrown/caught
- left and right hands throw alternatively

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A siteswap is a sequence of numbers indicating the height of the throw.

Example : "441441441441..."

Time	0	1	2	3	4	5	5	7	8	9	10	11
Hand	R	L	R	L	R	L	R	L	R	L	R	L
Height	4	4	1	4	4	1	4	4	1	4	4	1

 \rightarrow Siteswap "441"

Throw description

Fundamental siteswap pattern	Description
"0"	Empty hand
"1"	Direct pass to the other hand
"2"	Momentary hold
"3"	Low pass to the other hand
"4"	Medium pass to the same hand
"5"	High pass to the other hand

Two theorems

Theorem (Validity of a sequence)

The siteswap " $a_1 \dots a_k$ " is valid if the mapping

$$\mathbb{Z}/k\mathbb{Z} \longrightarrow \mathbb{Z}/k\mathbb{Z}$$
$$i \longmapsto a_i + i \mod k$$

is a bijection.

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Theorem

If the siteswap " $a_1 \dots a_k$ " is valid, then

$$B = \frac{1}{k} \sum_{i=1}^{k} a_i,$$

where B is the number of balls needed to juggle the siteswap.

Examples

Examples of invalid siteswaps :

 \rightarrow "214", "432", \ldots

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 \rightarrow "214", "432", \ldots

Examples of valid siteswaps :

 \rightarrow "3", "51", "53", \ldots

Thank you for your attention !

Siteswap simulator: https://www.jugglingedge.com

